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Intro to Derivatives Practice Quiz

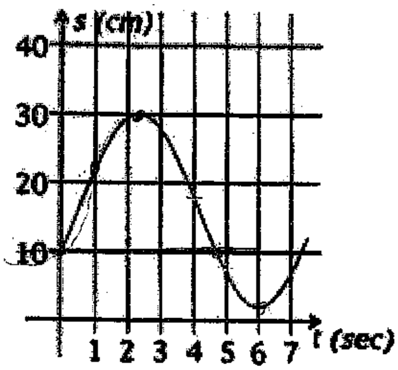
1. The figure shows the position versus time curve for a certain particle moving along a straight line. **Estimate** each of the following from the graph.

a) The average velocity over the interval $0 \le t \le 4.6$

b) The value of t at which the instantaneous velocity is zero

c) The values of t at which the instantaneous velocity is maximum; minimum

d) The instantaneous velocity when $t = 5$ seconds



a) 0

b) $\approx t = 2.3 + t = 6$

c) — Don't worry

d) — about these...

2. Let $f(x) = \frac{1}{x^2}$.

a) Find the average rate of change of y with respect to x over the interval [2,3].

b) Find the instantaneous rate of change of y with respect to x at the point $x = 2$.

a) $\left.\begin{matrix}(2, \frac{1}{4}) \\ (3, \frac{1}{9})\end{matrix}\right\} \Rightarrow \frac{\frac{1}{9} - \frac{1}{4}}{3-2} = \frac{\frac{4}{36} - \frac{9}{36}}{1} = \frac{-5}{36}$

b) $f'(2) = \lim_{x_1 \to 2} \frac{\frac{1}{x_1^2} - \frac{1}{4}}{x_1 - 2} = \lim_{x_1 \to 2} \frac{-\cancel{(x_1-2)}(x+2)}{4x^2} \cdot \frac{1}{\cancel{(x_1-2)}} = \lim_{x_1 \to 2} \frac{-(x+2)}{4x^2} = \frac{-4}{16} = \boxed{\frac{-1}{4}}$

2b) $\frac{1}{x^2} - \frac{1}{4} = \frac{4 - x^2}{4x^2} = \frac{-(x^2-4)}{4x^2} = \frac{-(x-2)(x+2)}{4x^2}$

3. Find the slope of the graph of f at the x-value specified by the given x_0.

$$f(x) = x^4 - 2 \qquad x_0 = 6$$

$f'(6) = \lim_{x_1 \to 6} \frac{x_1^4 - 2 - 1294}{x_1 - 6} = \lim_{x_1 \to 6} \frac{x^4 - 1296}{x - 6} = \lim_{x_1 \to 6} \frac{(x^2-36)(x^2+36)}{x-6} = \lim_{x_1 \to 6} \frac{\cancel{(x-6)}(x+6)(x^2+36)}{\cancel{(x-6)}} = (6+6)(6^2+36)$

$f'(6) = 864$

4. Use the definition of the derivative to calculate $f'(x)$ if $f(x) = 3x^2 - x$ and find the equation of the tangent to the graph of f at $x = 1$.

$3(1) - 1$

$f'(x) = \lim_{w \to x} \frac{3w^2 - w - (3x^2 - x)}{w - x} = \lim_{w \to x} \frac{\cancel{(w-x)}[3(w+x) - 1]}{\cancel{(w-x)}} = 3(x+x) -$

$3w^2 - 3x^2 - w + x$

$3(w-x)(w+x) - (w-x)$

$w - x[3(w+x) - 1]$

$(1, 2)$ $\boxed{f'(x) = 6x - 1}$ f'(x)

$\boxed{f'(1) = 5}$ $\boxed{y = 5x - 3}$ slope of tangent line

$y - 2 = 5(x - 1)$

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5. Use the definition of the derivative to find the derivative of each function with respect to x.

a) $f(x) = -4x - 2$

$f'(x) = \lim_{w \to x} \frac{-4w-2-(-4x-2)}{w-x} = \lim_{w \to x} \frac{-4(w-x)}{(w-x)}$

$\boxed{f'(x) = -4}$

b) $f(x) = -3x^2 + 4$

$f'(x) = \lim_{w \to x} \frac{-3w^2+4-(-3x^2+4)}{(w-x)}$

$= \lim_{w \to x} \frac{-3(w^2-x^2)}{(w-x)} = \lim_{w \to x} \frac{-3\cancel{(w-x)}(w+x)}{\cancel{(w-x)}}$

$= -3(2x) = \boxed{-6x}$

c) $y = 3x^2 + 3x + 3$

$\frac{dy}{dx} = \lim_{w \to x} \frac{3w^2+3w+3-(3x^2+3x+3)}{(w-x)}$

$= \lim_{w \to x} \frac{\cancel{(w-x)}[3(w+x)+3]}{\cancel{(w-x)}} = 3(x+x)+3)$

$\boxed{= 6x+3}$

d) $f(x) = \sqrt{4x-5}$

$f'(x) = \lim_{w \to x} \frac{\sqrt{4w-5}-\sqrt{4x-5}}{w-x} \cdot \frac{(\sqrt{4w-5}+\sqrt{4x-5})}{(\sqrt{4w-5}+\sqrt{4x-5})}$

$= \lim_{w \to x} \frac{4\cancel{(w-x)}}{\cancel{(w-x)}\sqrt{4w-5}+\sqrt{4x-5}}$

$= \frac{4}{\sqrt{4x-5}+\sqrt{4x-5}} = \boxed{\frac{4}{2\sqrt{4x-5}}}$ or $\frac{2}{\sqrt{4x-5}}$

e) $y = \frac{-2}{2x-1}$

$\frac{dy}{dx} = \lim_{w \to x} \frac{\frac{-2}{2w-1} - \frac{-2}{2x-1}}{w-x}$

$\lim_{w \to x} \frac{\frac{-4x+4w}{(2w-1)(2x-1)}}{w-x}$

$\lim_{w \to x} \frac{4\cancel{(w-x)}}{(2w-1)(2x-1)} \cdot \frac{1}{\cancel{(w-x)}} = \boxed{\frac{4}{(2x-1)^2}}$

e)

$\frac{-2(2x-1)+2(2w-1)}{(2w-1)(2x-1)}$

$\frac{-4x+2+4w-2}{(2w-1)(2x-1)}$

$\frac{-4x+4w}{(2w-1)(2x-1)}$

c) $3w^2-3x^2+3w-3x$

$3(w-x)(w+x)+3(w-x)$

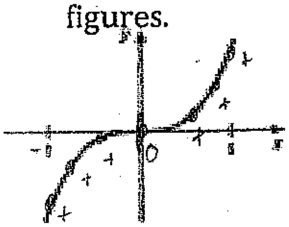
$(w-x)[3(w+x)+3]$

d) $4w-5-(4x-5)$

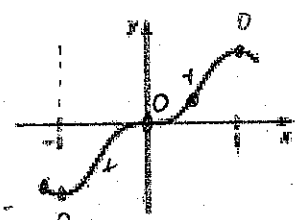
$4w-5-4x+\cancel{5}$

$4w-4x = 4(w-x)$

6. Match the functions (a)-(d) in first set of figures with the derivative functions (A)-(D) in the next set of figures.



(a)



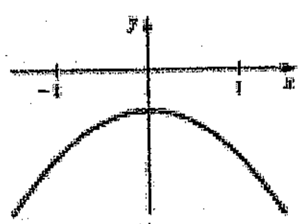
(b)

$a \to D$

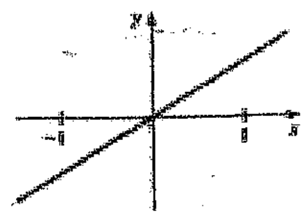
$b \to C$

$c \to B$

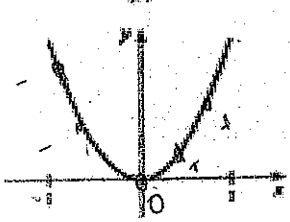
$d \to A$



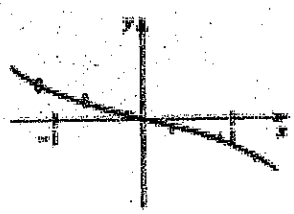
(A)



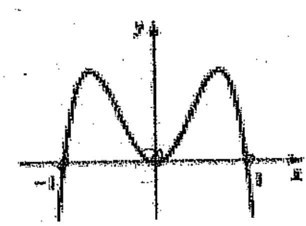
(B)



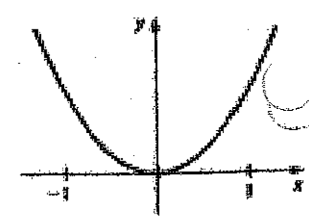
(c)



(d)



(C)



(D)