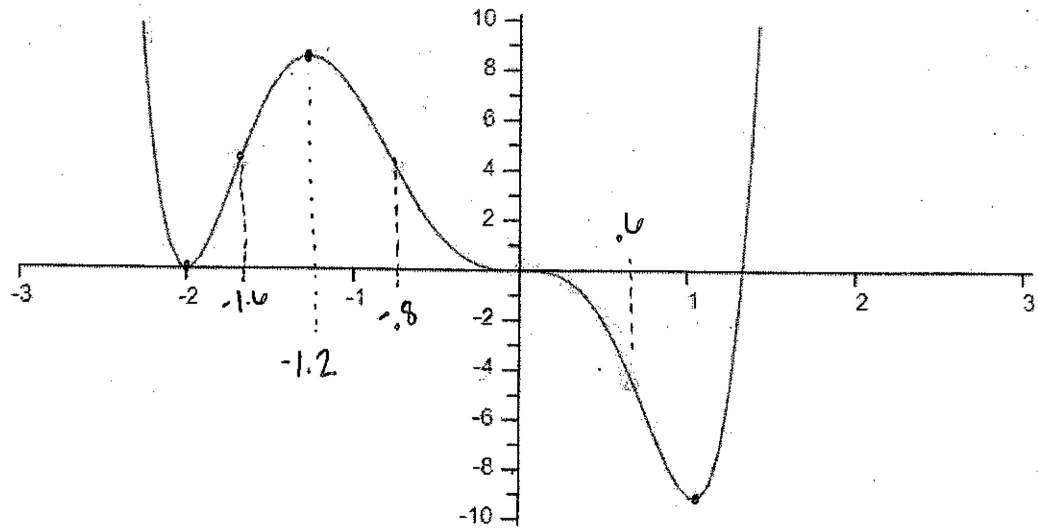
~~Intro~~ Greer Calculus

Name: ______________
Date: ______ Hour: ___



1. **Identify the critical points.**

$x = -2$ $\quad x \approx -1.2$ $\quad x = 0$ $\quad x = 1$

2. **State whether the critical points are relative maxima, relative minima, absolute maxima, absolute minima, or neither.**

$x = -2$: Rel. min.
$x \approx -1.2$: Rel. max
$x = 0$: neither
$x = 1$: abs. minimum

3. **Determine the interval where the function increases and decreases.**

$(-\infty, -2)$ decreasing
$(-2, -1.2)$ increasing
$(-1.2, 1)$ decreasing
$(1, \infty)$ increasing

4. **Determine the intervals on which the function is concave up and concave down.**

$(-\infty, -1.6)$: con. up
$(-1.6, -.8)$: con. down
$(-.8, 0)$ con. up
$(0, .6)$ con. down
$(.6, \infty)$ con. up

5. **Determine the points of inflection.**

$x = -1.6$
$x = -.8$
$x = 0$
$x = .6$

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Calcul

Equation 1: Let $f(x) = 20 + 6x^2 - x^3$

6. Find the interval of increasing/decreasing. 1. ____________

$f'(x) = 12x - 3x^2$

$0 = 3x(4-x)$

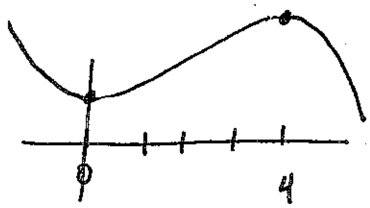
$x = 0,\ x = 4$

	$3x$	$4-x$	$f'(x)$	
-1 $(-\infty, 0)$	$-$	$+$	$-$	decreasing
1 $(0, 4)$	$+$	$+$	$+$	increasing
5 $(4, \infty)$	$+$	$-$	$-$	decreasing

7. State which x-values are the critical points. 2. $x=0,\ x=4$

$x=0$ $x=4$

8. Sketch a graph to determine whether the critical points are relative maxima/relative minima/ or neither.



3. $x=0$ R. min
$x=4$ R. max

9. Find the interval of concave up/concave down. 4. ____________

$f'(x) = 12x - 3x^2$

$f''(x) = 12 - 6x$

$0 = 6(2-x)$

$x = 2$

	~~$x-2$~~ $2-x$	$f''(x)$	
0 $(-\infty, 2)$	$+$	$+$	concave up
3 $(2, \infty)$	$-$	$-$	concave down

10. State which x-values are the points of inflection. 5. $x = 2$

↑

they are only points of inflection if it goes from concave up to concave down (or vice versa)

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Equation 2: Let $f(x) = x^5 - 4x^4 + 4x^3$

11. Find the interval of increasing/decreasing. 1. ____________________

$f'(x) = 5x^4 - 16x^3 + 12x^2$

$0 = x^2(5x^2 - 16x + 12)$

$x = 0$

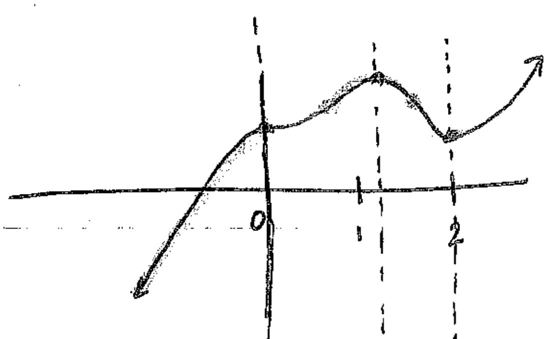
$x = \frac{16 \pm \sqrt{(-16)^2 - (4 \cdot 5 \cdot 12)}}{10}$

$x = 1.2 \quad x = 2$

	x^2	$x-1.2$	$x-2$	$f'(x)$
-1 $(-\infty, 0)$	+	−	−	+ inc
1 $(0, 1.2)$	+	−	−	+ inc.
1.5 $(1.2, 2)$	+	+	−	− dec.
3 $(2, \infty)$	+	+	+	+ inc.

12. State which x-values are the critical points. 2. $x = 0, 1.2, 2$

13. Sketch a graph to determine whether the critical points are relative maxima/relative minima/ or neither.



3. $x = 0$ neither
$x = 1.2$ R. max
$x = 2$ R. min.

14. Find the interval of concave up/concave down. 4. ____________________

$f''(x) = 20x^3 - 48x^2 + 24x$

$0 = 4x(5x^2 - 12x + 6)$

$x = 0$

$x = \frac{12 \pm \sqrt{(-12)^2 - (4 \cdot 5 \cdot 6)}}{10}$

$x = .71 \quad x = 1.7$

	$4x$	$x-.71$	$x-1.7$	$f''(x)$
-1 $(-\infty, 0)$	−	−	−	−
.5 $(0, .71)$	+	−	−	+
1 $(.71, 1.7)$	+	+	−	−
2 $(1.7, \infty)$	+	+	+	+

15. State which x-values are the points of inflection. 5. ____________________

$x = 0, \ x = .71, \ x = 1.7$

Calculus

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Let $f(x) = x^4 + 8x^3 + 2x^2$

16. Find the absolute maxima and absolute minima on the closed interval $[-2, -10]$.

$f'(x) = 4x^3 + 24x^2 + 4x$

$0 = 4x(x^2 + 6x + 1)$

$x = 0$

$x = \frac{-6 \pm \sqrt{6^2 - (4 \cdot 1 \cdot 1}}{2}$

$x = \cancel{-}.17 \quad x = -5.8$

x	f(x)
-2	-40
-10	2200
0	0
-5.8	-362

☆ abs. max is 2200 @ x = -10

☆ abs. min is -362 @ x = -5.8

17. Find the absolute maxima and absolute minima on an open interval.

* no abs. max (even / pos. poly) ↑ ↗

* abs. min is -362 @ x = -5.8

Let $f(x) = 3x^3 + 9x^2$

18. Find the absolute maxima and absolute minima on the closed interval $[-5, 5]$.

$f'(x) = 9x^2 + 18x$

$0 = 9x(x + 2)$

$x = 0, -2$

x	f(x)
0	0
-2	12
5	600
-5	-150

☆ abs. max is 600 @ x = 5

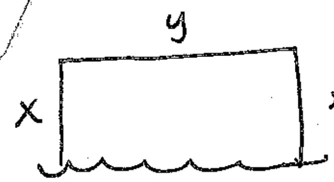
☆ abs. min is -150 @ x = -5

19. Find the absolute maxima and absolute minima on an open interval.

* no abs max. }
* no abs. min } (odd deg. poly)

↗ ↙

ner has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. ds no fence along the river. What are the dimensions of the field that has the largest area?



$2x+y=2400$

$A=x\cdot y$

$A=x(-2x+2400)$

$[0,1200]$

$A=-2x^2+2400x$

$A'=-4x+2400$

$0=-4(x-600)$

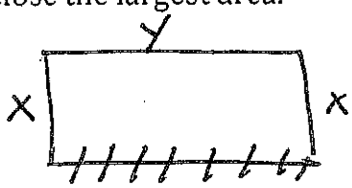
$x=600$

x	y
0	0
1200	0
600	720,100

$x=600$

$y=2400-(1200)=1200$

21. We need to enclose a field with a rectangular fence. We have 500 ft of fencing material and a building on one side of the field and so won't need any fencing. Determine the dimensions of the field that will enclose the largest area.



$2x+y=500$

$A=xy$

$A=x(500-2x)=500x-2x^2$

$[0,250]$

$A'=500-4x$

$0=4(125-x)$

$x=125$

x	y
0	0
0	0
125	31,250

$x=125$

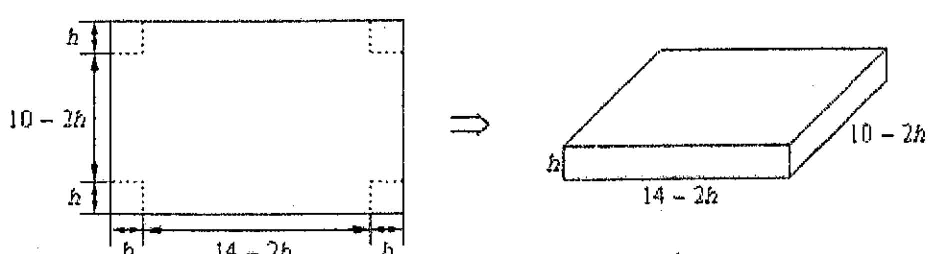
$y=500-[250]=250$

22. We have a piece of cardboard that is 14 in by 10 in and we're going to cut out the corners as shown be and fold up the sides to form a box, also shown below. Determine the height of the box that will give a maximum volume.

<u>max:</u> $V=(10-2h)(14-2h)h$

5 7 0

$[0,5]$



$V=140h-48h^2+4h^3$

$V'=140-96h+12h^2$

$h=\dfrac{96\pm\sqrt{96^2-4\cdot12\cdot14}}{2\cdot12}$

$h=1.9,\ \cancel{6.1}$

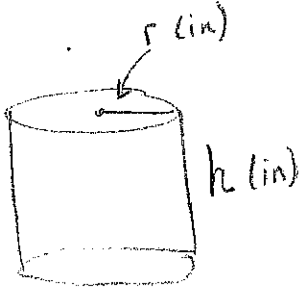
x	y
0	0
5	0
1.9	120

23. What are the dimensions of an aluminum can that holds $40 in^3$ of juice and uses the least material. Assume cylinder is capped on both ends.

$$V = \pi r^2 h$$

$$SA = A = 2\pi r^2 + 2\pi r h$$

(1)



(2) $A = 2\pi r^2 + 2\pi r h$

(3) $40 = \pi r^2 h$

(4) $h = \dfrac{40}{\pi r^2} \Rightarrow A = 2\pi r^2 + 2\pi \cancel{r} \left(\dfrac{40}{\pi r^{\cancel{2}}}\right)$

$$A = 2\pi r^2 + \frac{80}{r}$$

(5) $(0, \infty)$

(6) $A' = 4\pi r - \dfrac{80}{r^2}$

$$0 = 4\pi r - \frac{80}{r^2} = \frac{4\pi r^3 - 80}{r^2} = 0$$

$$4\pi r^3 - 80 = 0$$

$$r^3 = \frac{80}{(4\pi)}$$

$$\boxed{r \approx 1.85}$$

$$\boxed{h = \frac{40}{\pi (1.85)^2} \approx 3.7}$$

$$V = \pi (1.85)^2 (3.7)$$

$$V = \approx 40 in^3$$