Kuta Software - Infinite Calculus

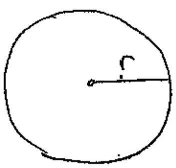
Name: Key

Related Rates

Date________________ Period____

Solve each related rate problem.

1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?



$\frac{dr}{dt} = 4 \text{ cm/min}$

$\left.\frac{dA}{dt}\right|_{r=5} = ?$

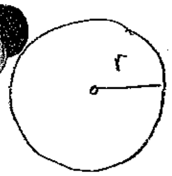
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(5)(4)$

$\frac{dA}{dt} = 40\pi \text{ cm./min}$

2) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 9π m²/min. How fast is the radius of the spill increasing when the radius is 10 m?



$\frac{dA}{dt} = 9\pi \text{ m}^2/\text{min}$

$\left.\frac{dr}{dt}\right|_{r=10} = ?$

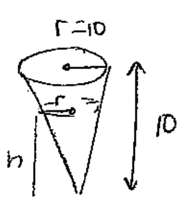
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$9\pi = 2\pi(10)\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{9\cancel{\pi}}{20\cancel{\pi}} = \frac{9}{20} \text{ m/min}$

3) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?



$\frac{dh}{dt} = 2 \text{ cm/sec}$

$\left.\frac{dV}{dt}\right|_{h=8} = ?$

$\frac{r}{h} = \frac{10}{10}$

$\frac{10r}{10} = \frac{10h}{10}$

$r = h$

$V = \frac{1}{3}\pi r^2 \cdot h$

$V = \frac{1}{3}\pi (h)^2 \cdot h$

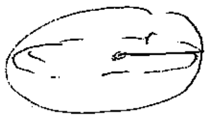
$V = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \frac{1}{\cancel{3}}\pi \cdot \cancel{3}h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi(64)(2)$

$\frac{dV}{dt} = 128\pi \text{ cm}^3/\text{s}$

4) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{2}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?



$\frac{dr}{dt} = \frac{2}{r}$ cm/sec

$\left.\frac{dV}{dt}\right|_{r=4}$

$V = \frac{4}{3}\pi r^3$

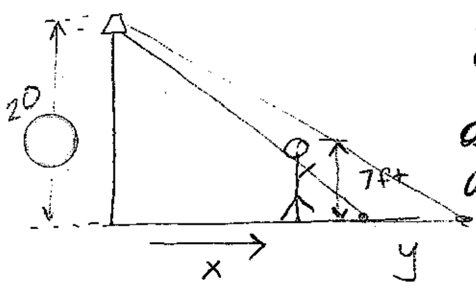
$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{2}{r} = \frac{2}{4}$

$\frac{dV}{dt} = 4\pi 4^2 \cdot \left(\frac{2}{4}\right)$

$\frac{dV}{dt} = 32\pi\ cm^3/sec$

~~5)~~ A 7 ft tall person is walking away from a 20 ft tall lamppost at a rate of 5 ft/sec. Assume the scenario can be modeled with right triangles. At what rate is the length of the person's shadow changing when the person is 16 ft from the lamppost?



$\frac{dx}{dt} = 5$ ft./sec

$\frac{dy}{dt} = ?$

$\frac{x+y}{20} = \frac{y}{7}$

$7x + 7y = 20y$

$7x = 13y$

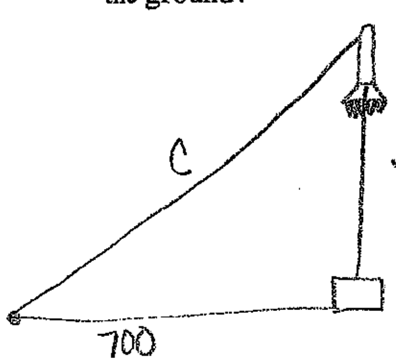
$7 \cdot \frac{dx}{dt} = 13\frac{dy}{dt}$

$\frac{7}{13} \cdot \frac{dx}{dt} = \frac{dy}{dt}$

$\frac{7}{13} \cdot \frac{5}{1} = \frac{dy}{dt}$

$\boxed{\frac{35}{13}\ ft/sec = \frac{dy}{dt}}$

6) An observer stands 700 ft away from a launch pad to observe a rocket launch. The rocket blasts off and maintains a velocity of 900 ft/sec. Assume the scenario can be modeled as a right triangle. How fast is the observer to rocket distance changing when the rocket is 2400 ft from the ground?



$\frac{dy}{dx} = 900$ ft/sec

$\left.\frac{dc}{dx}\right|_{y=2400}$

$700^2 + y^2 = c^2$

$0 + 2y\frac{dy}{dt} = 2c\frac{dc}{dt}$

$2(2400)900 = 2(2500)\frac{dc}{dt}$

$\frac{dc}{dt} = 864$ ft./sec.

$700^2 + 2400^2 = c^2$

$c = 2500$